Jom Skor A+
SPM 2021

MATEMATIK TAMBAHAN

Permutation & Combination

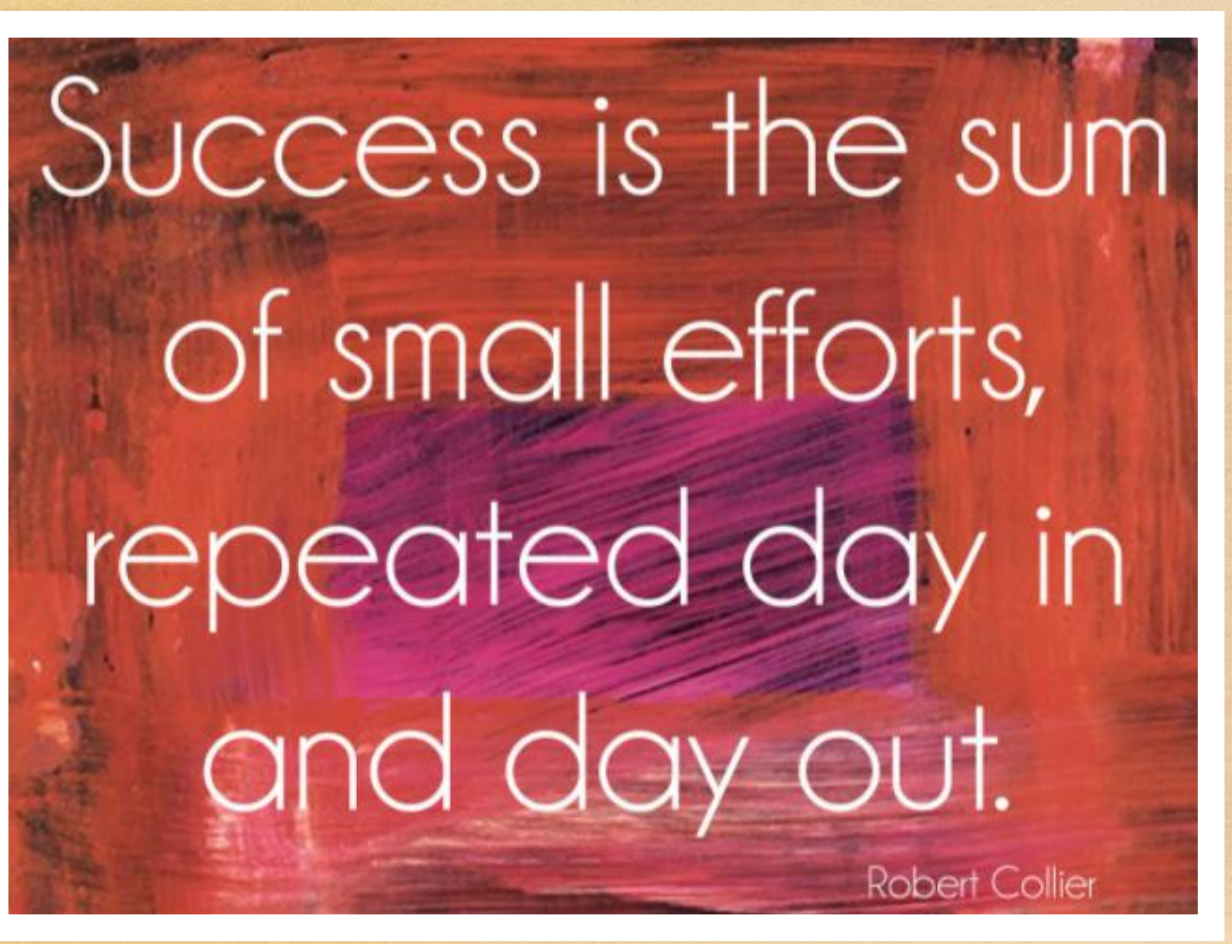
5 November 2021 | Friday

3.00 pm – 5.00 pm

https://bit.ly/CgSuhailaPermutationandCombination

Cg. Suhaila Sulong
MRSM Tun Dr. Ismail

Anjuran Unit Matematik
Bahagian Pendidikan Menengah


Success is the sum
of small efforts,
repeated day in
and day out.
Robert Collier


FAILURE DOESN'T MEAN THE
GAME IS OVER,
IT MEANS TRY AGAIN WITH
EXPERIENCE

DOA BELAJAR

اَللّٰهُمَّ افْتَحْ عَلَيْنَا حِكْمَتَكَ وَانْشُرْ عَلَيْنَا
مِنْ خَزَآئِنِ رَحْمَتِكَ يَآ أَرْحَمَ الرَّاحِمِيْنَ

WAHAI TUHAN KAMI, BUKALAH KE ATAS KAMI HIKMAH-MU DAN
BENTANGKAN KE ATAS KAMI GEDUNG RAHMAT-MU.
WAHAI TUHAN YANG MAHA PENGASIH

http://muhassigns.blogspot.com

DOA PENERANG HATI

رَبِّ اشْرَحْ لِى صَدْرِى وَيَسِّرْلِى اَمْرِى
وَاحْلُلْ عُقْدَةً مِنْ لِسَانِى يَفْقَهُوْا قَوْلِى

Maksudnya:
Wahai Tuhanku! Lapangkanlah bagiku dadaku dan mudahkanlah bagiku urusanku dan bukalah simpulan dari lidahku supaya mereka faham ucapanku.

What we need to know about PERMUTATION?

ORDER OF ARRANGEMENT IS IMPORTANT

1. MULTIPLICATION RULE
2. Permutations for n different objects
3. Permutations of n different objects, taking r objects each time + (Circular Permutations)
4. Permutations for n objects involving identical objects + (Circular permutations)
5. Permutations with certain conditions

1. MULTIPLICATION RULE

4.1 : Multiplication Rule

Multiplication rule states that if an event can occur in m ways and a second event can occur in n ways, then both events can occur in m × n ways.

QUESTION 1 :

A shop sells 4 different flavours of yoghurt drink in 3 different sizes. Nadia wants to buy a bottle of yoghurt drink, find the number of possible choices.

Flavours, $m = 4$ Sizes, $n = 3$

Number of possible choices = $m \times n$

$= 4 \times 3$

$= 12$

4.1 : Multiplication Rule

QUESTION 2 :

There are 3 different routes to travel from location A to location B and 2 different routes to travel from location B to location C. Find the number of ways a person can travel from location A to location C via location B.

Number of routes from A to B, $m = 3$

Number of routes from B to C, $n = 2$

Number of ways a person can travel from location A to location C via location B $= m \times n$

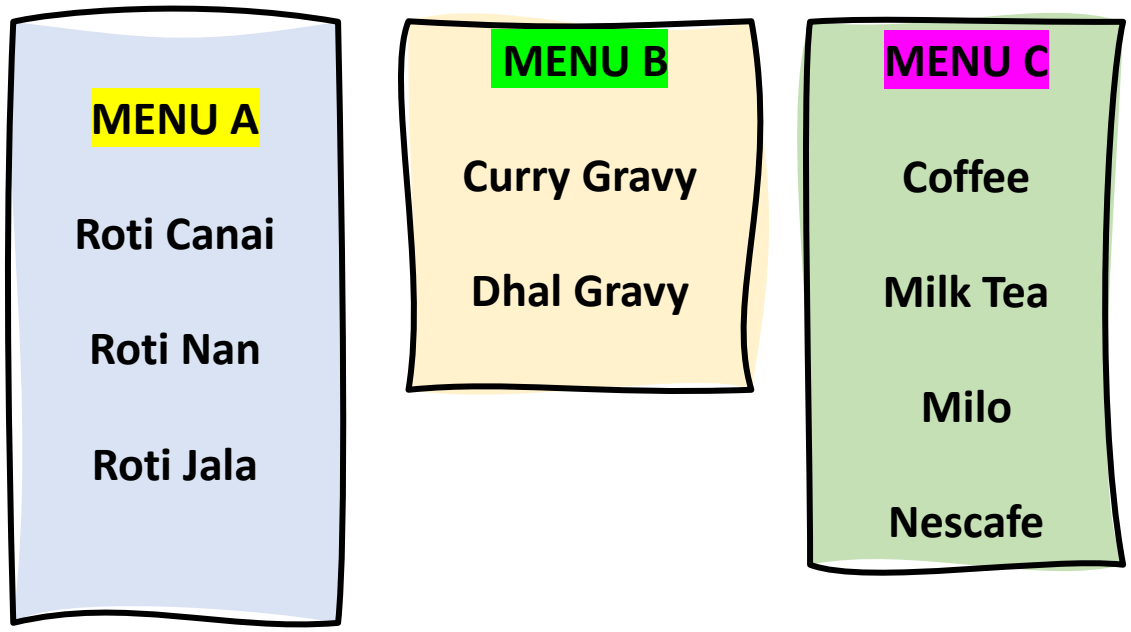
$= 3 \times 2$

$= 6$

4.1 : Multiplication Rule

QUESTION 3 :

A Kopitiam shop offer a breakfast set. The menu are as in the diagram below. Find the number of breakfast set that can be chosen from the menu if customer need to choose only one from each Menu A, Menu B and Menu C.



Menu A = 3

Menu B = 2

Menu C = 4

Number of breakfast set that can be chosen = A × B × C

= 3 × 2 × 4

= 24

2. Permutations for n different objects

4.2 : Permutations For n Different Objects

The number of permutations of n objects is given by n!, where

$$n! = {}^nP_n$$

$$= n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1$$

QUESTION 4 :

Without using a calculator, find the value of each of the following.

a) 6! b) $\frac{7!}{3!}$ c) $\frac{6!}{3!2!}$

a) $6!$

$= 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 720$

b) $\frac{7!}{3!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1}$

$= 7 \times 6 \times 5 \times 4$

$= 840$

c) $\frac{6!}{3!2!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 2 \times 1}$

$= \frac{6 \times 5 \times 4}{2 \times 1}$

$= 60$

4.2 : Permutations For n Different Objects

QUESTION 5 :

Arina buys 3 different types of potted plants. Find the number of ways she can arrange the plants in a row.

3 ways

2 ways

1 ways

Number of ways she can arrange the plants in a row

$= 3 \times 2 \times 1$

$= 6$

$^3P_3$

Ways to arrange the potted plants in a row

4.2 : Permutations For n Different Objects

QUESTION 6 :

Find the number of 4-digit numbers that can be formed from the digits 1, 3, 7 and 8 , without repetition of digits

$$\_\_\_\_ \quad \_\_\_\_ \quad {}^{4}P_{4} \quad \_\_\_\_ \quad \_\_\_\_$$

Number of 4-digit numbers that can be formed

$= {}^{4}P_{4}$

$= 24$

4.2 : Permutations For n Different Objects

QUESTION 7 :

Find the number of different arrangement of this nine cards if there is no restrictions

Number of different arrangement of this nine cards

$= {}^{9}P_{9}$

$= 362880$

Permutations Of N Objects In A Circle

A B C

From Question 5,
Number of ways to arrange
3 potted plant

$= {}^{3}P_{3}$

$= 3 \times 2 \times 1$

$= 6$

A B C | B A C | C A B

A C B | B C A | C B A

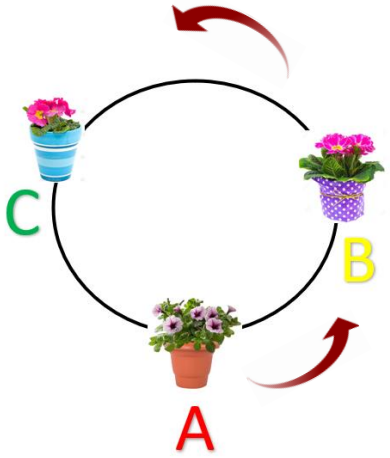
These Are Graphical Representation Of 6 Ways To Arrange 3 Potted Plant

HOW TO ARRANGE THESE 3 POTTED PLANT IN A CIRCULAR SHAPE?

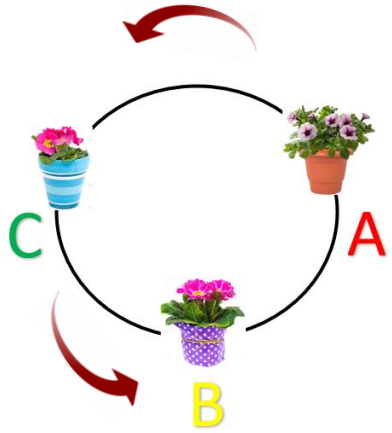
How many ways to arrange the plotted plant in a CIRCULAR SHAPE?

A B C | B A C | C A B

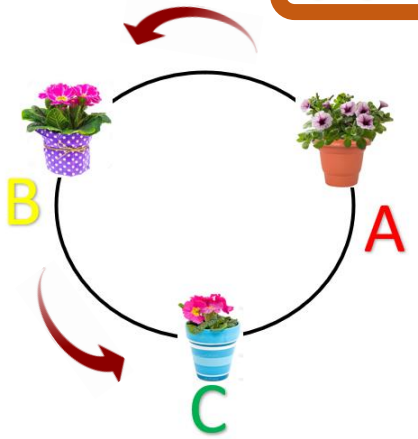
A C B | B C A | C B A



A – B – C

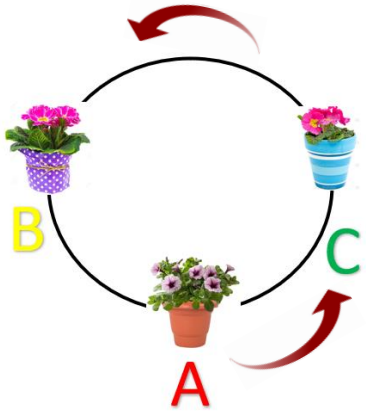


A – C – B

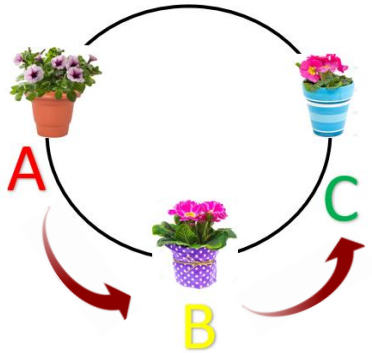


A – B – C

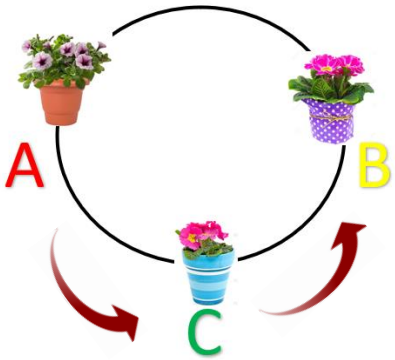
LOOK AT THE ANTICLOCKWISE ARRANGEMENT



A – C – B



A – B – C



A – C – B

WE HAVE 2 DIFFERENT ARRANGEMENTS

A – B – C

A – C – B

IN GENERAL, THE PERMUTATION OF N OBJECTS IN A CIRCLE IS GIVEN BY

$$= \frac{n!}{n}$$

$$= \frac{n \times (n-1) \times (n-2) \times (n-3) \ldots}{n}$$

$$= (n-1) \times (n-2) \times (n-3) \ldots$$

$$= (n-1)!$$

LETS CHECK!!

n = 3
Because we have 3 potted plants

→

$$\frac{3!}{3}$$

$$= (3-1)!$$

$$= 2$$

4.2 : Permutations For n Different Objects (Circle)

QUESTION 8 :

Determine the numbers of ways to arrange eight pupils to sit at a round table.

n = 8

Numbers of ways to arrange eight pupils = (n – 1)!

= (8 – 1)!

= 7!

= 5040

PERMUTATIONS FOR N DIFFERENT OBJECTS ARRANGE IN A CIRCLE

ANTICLOCKWISE = CLOCKWISE

Turn around to right hand

C

D

B

B

D

A

Left hand

A → B → C → D

A

Right hand

A → B → C → D

A B C | B A C | C A B

A C B | B C A | C B A

WE HAVE 2 DIFFERENT ARRANGEMENTS

A – B – C

A – C – B

Circular Permutation

Both Anti Clockwise

Left hand

A – B – C

Right hand

A – C – B

FORMULA FOR CIRCULAR PERMUTATION

$= (n - 1)!$

Circular permutation involving necklace or bracelet $= \frac{(n-1)!}{2}$

Double counting

A → B → C

Clockwise for the right hand

4.2 : Permutations For n Different Objects (Circle)

QUESTION 9 :

Determine the number of ways to arrange nine gemstones with different colour to form a chain.

Number of ways to arrange nine gemstones

$$= \frac{(n-1)!}{2}$$

$$= \frac{(9-1)!}{2}$$

$$= \frac{8!}{2}$$

$$= 20160$$

The beads are arranged in a circle. It is found that the clockwise and anticlockwise arrangements look the same

3. Permutations For n Different Objects ,taking r objects each time

4.3 : Permutations For n Different Objects ,Taking r Objects Each Time

The number of permutations of
n objects taking r each time is given by

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$, where r ≤ n

QUESTION 10 :

Without using a calculator, find the value of ${}^{5}P_{3}$

$${}^{5}P_{3} = \frac{5!}{(5-3)!}$$

$$= \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1}$$

$$= 5 \times 4 \times 3$$

$$= 60$$

4.3 : Permutations For n Different Objects ,Taking r Objects Each Time

QUESTION 11 :

n = 6

There are 6 parking lots in a row in front of a shop. Find the number of ways 4 drivers can park their cars in front of the shop.

r = 4

Number of ways 4 drivers can park their cars in front of the shop

$= {}^{6}P_{4}$

$= \frac{6!}{(6-4)!}$

$= 360$

4.3 : Permutations For n Different Objects ,Taking r Objects Each Time

QUESTION 12 :

n = 6

r = 4

Given six digits, 1, 2, 3, 4, 5 and 6. Find the number of 4-digit numbers that can be formed from the 6 digits if repetition of digits is not allowed

The number of 4 digits numbers that can be formed

$= {}^{6}P_{4}$

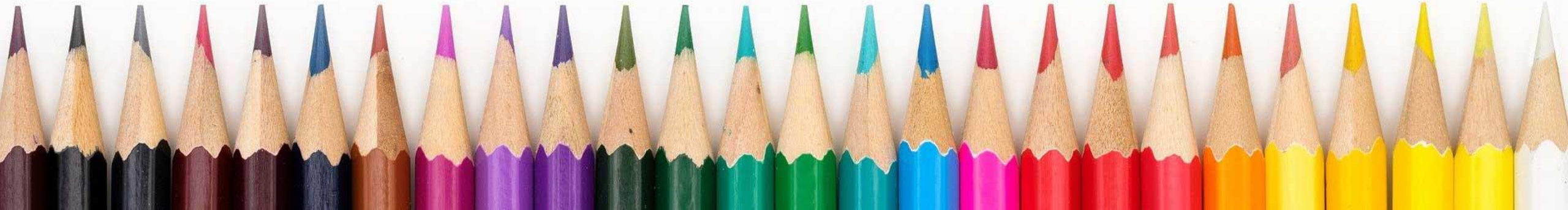
$= \frac{6!}{(6-4)!}$

$= 360$

3. Permutations For n Different Objects ,taking r objects each time (Circle)

1. The number of permutations for n different objects taking r objects each time and arranged in a circle is given by $\frac{{}^{n}P_{r}}{r}$

2. A permutation of an object in a circle where clockwise and anticlockwise arrangements are the same, then the number of ways is as follows $\frac{{}^{n}P_{r}}{2r}$



4.3 : Permutations For n Different Objects ,Taking r Objects Each Time (Circlce)

QUESTION 13 :

n = 10

An employee at a restaurant needs to arrange 10 plates on a round table but the table can only accommodate 6 plates. Find the number of ways to arrange the plates.

r = 6

Number of ways to arrange the plates

$$= \frac{{}^{10}P_6}{6}$$

Need to be divided by 6 because 6 plates is repeating 6 times

$$= 25200$$

What about BRACELET ? WHERE IT DID NOT MATTER EITHER CLOCKWISE OR ANTI CLOCKWISE ??

LETS LOOK AND THE NEXT QUESTIONS.

4.3 : Permutations For n Different Objects ,Taking r Objects Each Time (Circlce)

QUESTION 14 :

n = 10

Safiyyah bought 10 beads of different colours from Handicraft Market in Kuching and she intends to make a bracelet. Safiyyah realises that the bracelet requires only 6 beads. How many ways are there to make the bracelet?

r = 6

No of ways to make the bracelet

$$= \frac{{}^{10}P_6}{6\,(2)}$$

$$= 12600$$

A permutation of an object in a circle where clockwise and anticlockwise arrangements are the same.

4. Permutations For n Objects Involving Identical Objects

4.4 : Permutations For n Objects Involving Identical Objects

The number of permutations for n objects involving identical objects is given by

$$P = \frac{n!}{a!b!c!}$$

where a, b and c, … are the number of identical objects for each type.

QUESTION 15 :

Calculate the number of ways to arrange all the letters of the word 'ASSESSMENT'.

The number of ways to arrange

$$= \frac{10!}{4!2!}$$

$$= 75600$$

The word 'ASSESMENT' consists of 10 letters with 4 identical letter 'S' and 2 identical letter 'E'.

4.4 : Permutations For n Objects Involving Identical Objects

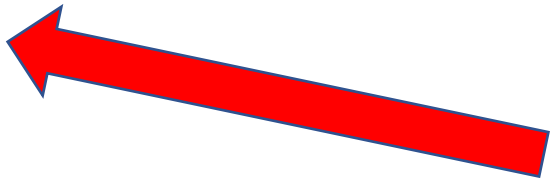
QUESTION 16 :

There are 5 blue crayons and 3 red crayons in a container. Find the number of ways to arrange all the crayons in one line.

The number of ways to arrange

$$= \frac{8!}{5! \times 3!}$$

$$= 56$$



Identical

5 blue crayons = 5 !

3 red crayons = 3 !

5. Permutations With Certain Conditions

4.5 : Solving Problems Involving Permutations With Certain Conditions

QUESTION 17 :

Calculate the number of 3 digit odd numbers that can be formed from the digits 2,3, 4, 6 and 7 without repetitions.

The number of arrangements for the other 4 digits in two empty spaces is $^4P_2$

$^4P_2$ _____ _____ 3 OR 7

3 digit odd numbers have to end with a digit 3 or a digit 7

Because we have 2 choices to be put at 1 place, that is 3 or 7

The total number of three digit odd numbers

$= {}^2P_1 \times {}^4P_2$

$= 2 \times 12$

$= 24$

PEMBETULAN

4.5 : Solving Problems Involving Permutations With Certain Conditions

QUESTION 18:

3 women and 2 men are lining up to receive free drinks. Find the number of possible arrangements if two men are at both ends of the line

We consider Men 1 = M1 and Men 2 = M2

We have 2 situation

M1 _____ $^3P_3$ _____ M2 or

M2 _____ $^3P_3$ _____ M1

The number of arrangements for the other 3 women in three empty spaces is $^3P_3$

The number of possible arrangement

$= {}^3P_3 + {}^3P_3$

$= 6 + 6$

$= 12$

4.5 : Solving Problems Involving Permutations With Certain Conditions

QUESTION 19 :

6 female students and 5 male students are lining up for a photo session. Find the number of possible arrangements if the female students and male students have to be arranged alternately.

6	5	5	4	4	3	3	2	2	1	1

$6\times5\times4\times3\times2\times1$ $\qquad$ $5\times4\times3\times2\times1$

$${}^{6}P_{6} \times {}^{5}P_{5}$$

$$= 86400$$

4.5 : Solving Problems Involving Permutations With Certain Conditions

QUESTION 20 :

Calculate the number of different ways the letter from the word '*NUKLEAR*' can be arranged such that the vowels have to be side by side.

Vowels = U, E, A → Assume that we tied it together so it will become 1 unit → ${}^{3}P_{3}$ All the 3 vowels can interchange among themselves

${}^{5}P_{5}$

Because the vowels become 1 unit, therefore we need to consider to arrange 5 letter (N, K, L, R and [U,E,A])

The number of different ways

$= {}^{5}P_{5} \times {}^{3}P_{3}$

$= 720$

4.5 : Solving Problems Involving Permutations With Certain Conditions (Circle)

QUESTION 21 :

In an IB classroom, 5 students A, B, C, D and E need to be arranged at a round table. Find the number of ways to arrange all 5 students if 2 students A and B want to be seated together.

Student A and B are seated together
So they are regarded as one unit

← A and B

A and B can interchange among themselves → $^2P_2$

To arrange one unit of A and B and three others

$= (n – 1)!$

$= (4 -1)!$

$= 6$

The number of ways to arrange all 5 students

$= (4 – 1)! \times {}^2P_2$

$= 6 \times 2$

$= 12$

4.5 : Solving Problems Involving Permutations With Certain Conditions (Identical)

QUESTION 22 :

9 cards written with the letters from the word "COMMITTEE" are to be arranged in a row. If the vowels have to be arranged side-by-side, calculate the number of different arrangements

Vowels = O, I, E, E → Assume that vowels arranged side by side so it will become as 1 unit

Arrangement of vowel with identical letter 'E' $= \frac{4!}{2!}$

$$\frac{6!}{2! \times 2!}$$

← 2 identical letter 'M' and 2 identical letter 'T'.

Because the vowels become 1 unit, therefore we need to consider to arrange 6 letter
(C, M, M, T, T and [O,I,E,E])

The number of different arrangements

$= \frac{6!}{2! \times 2!} \times \frac{4!}{2!}$

$= 180 \times 12$

$= 2160$

4.5 : Solving Problems Involving Permutations With Certain Conditions

QUESTION 23 :

Calculate the number of four digit even number less than 5000 that can be formed from the digits 3, 4, 5, 6 and 9 without repetitions.

Less than 5000, digits that can be considered ▶ 3 or 4 _____ _____ _____ 4 or 6 ◀ Even number, digits that can be considered

Situation A

$$\underline{3}_{1} \times {}^{3}P_{2} \times \underline{4 \text{ or } 6}_{{}^{2}P_{1}} = 12$$

OR

Situation B

$$\underline{4}_{1} \times {}^{3}P_{2} \times \underline{6}_{1} = 6$$

No of ways
= 12 + 6
= 18

SPMRSM, SPM AND FORECAST QUESTION (PERMUTATION)

SPMRSM, SPM AND FORECAST QUESTIONS

QUESTION 24:

Given ${}^{n+2}P_3 = 30n$, find n value. (3 marks)

Formulae ${}^{n}P_r = \frac{n!}{(n-r)!}$

${}^{n+2}P_3 = \frac{(n+2)!}{(n+2-3)!} = 30n$

$\frac{(n+2)!}{(n+2-3)!} = 30n$ ✓

$\frac{(n+2)\times(n+1)\times(n)\times\cancel{(n-1)\times(n-2)\times(n-3)\times\cdots}}{\cancel{(n-1)\times(n-2)\times(n-3)\times\cdots}} = 30n$

$(n+2)\times(n+1)\times(n) = 30n$

$(n+2)\times(n+1) = 30$

$n^2 + 3n - 28 = 0$ ✓

$(n+7)(n-4) = 0$

$n = -7, n = 4$

$\therefore n = 4$ ✓

SPMRSM, SPM AND FORECAST QUESTIONS

QUESTION 25:

The diagram shows six cards of different letters.

P	U	T	E	R	A

Find the number of arrangements of all the letters if

(a) no condition is imposed,

(b) the letters P and A are not together. (4 marks)

(a)Number of arrangement

$= {}^{6}P_{6}$

$= 720$ ✓

(b) Number of arrangement

$= 720 - ({}^{5}P_{5} \times {}^{2}P_{2})$ ✓ ✓

$= 480$ ✓

Arrangement with no condition

Arrangement when P and A are side by side

SPMRSM, SPM AND FORECAST QUESTIONS

QUESTION 26:

The diagram shows seven cards of different letters.

S	U	A	S	A	N	A

Calculate the number of different ways to arrange all the cards in a row if

(a) there is no restriction.

(b) the vowels are always separated.

[3 marks]

(a) Number of different ways $= \frac{7!}{2!3!}$

Identical "S" = 2
and Identical "A" = 3

$= 420$

(b)

V C V C V C V

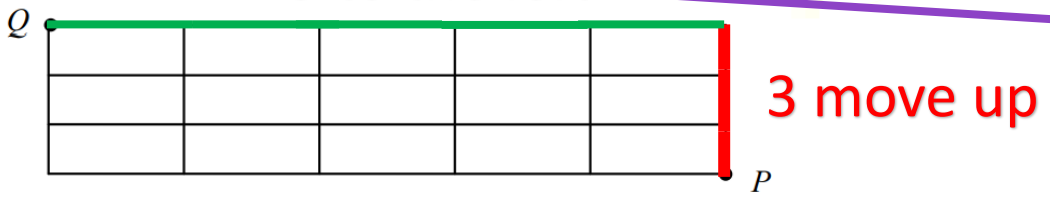
Number of different ways $= \frac{{}^4P_4 \times {}^3P_3}{2!3!}$

$= 12$

SPMRSM, SPM AND FORECAST QUESTIONS

QUESTION 27:

The diagram shows the routes for an object move from point P to point Q.



If the object can only move up or to the left, find the number of shortest routes for the object move from point P to point Q.

[2 marks]

5 to the left

3 move up

No. of movement

= 8

Number of shortest routes from P to Q

$$= \frac{8!}{3!5!}$$

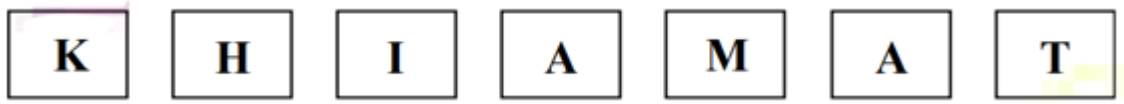
Identical "move up" = 3
and Identical "Left" = 5

= 56

SPMRSM, SPM AND FORECAST QUESTIONS

QUESTION 28:

Diagram shows seven letter cards.



A five – letter code is to be formed using five of these cards.
Find the number of different five-letter codes that can be formed.

TWO situation

There is 2 "A" but can ONLY use 1 (different five-letter)

With "A" : A,A ; K, H, I, M, T → $^{2}P_{1} \times {}^{5}P_{4} \times 5 = 1200$

"A" can be at any five places

OR

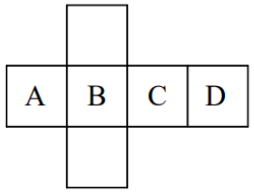
Without "A" : K, H, I, M, T → $^{5}P_{5} = 120$

Number of different five-letter codes = 1200 + 120
= 1320

SPMRSM, SPM AND FORECAST QUESTIONS

QUESTION 29:

The diagram shows the net of a cube formed by Atana.



She wants to colour the faces of cube which are labelled up with letter A, B, C and D with the different colours using the colour of rainbow. Find the number of ways to colour the four faces if

(a) red colour must be used. (*Ans* : 480)

(b) red , green and blue colours must be used, (*Ans* : 96)

[4 marks] [*Forecast*]

Rainbow = 7 colours

(a)

Possibility of red colour at A, B, C and D = 4

Number of ways = $4 \times {}^{6}P_{3}$

$= 480$

Remaining 6 colour to be put at 3 places

(b) Number of ways to colour the four faces

$= {}^{4}P_{3} \times {}^{4}P_{1} = 96$

4 places for arrangement of 3 colours

1 colour to be arranged from balanced 4 colour

SPMRSM, SPM AND FORECAST QUESTIONS

QUESTION 30:

The diagram shows two rows of chair. There are 4 seats at the first row, and 3 seats at the second row.

A group of people consisted of 2 children, 1 couple of husband-wife, and 4 adults. Find the number of ways of to arrange them in the 7 seats, if

(a) the couple of husband-wife and 2 children must sit in the same row, (*Ans* : 576)

(b) 2 children must sit in the same row, (*Ans* : 12960)

[5 marks] [***Forecast***]

(a) Number of ways = $4! \times {}^4P_3$

= 576

4! = husband & wife and 2 children at first row

4 adults to be seated at 3 chairs 2nd row

(b) Number of ways

= ${}^4P_2 \times {}^6P_5 + {}^3P_2 \times {}^6P_5$

= 12960

2 children arranged at first row

2 children arranged at 2nd row

SPMRSM AND SPM QUESTIONS

QUESTION 31:

Photograph 22, taken by a photographer shows a group of students in an MRSM.

Photograph 22

The photographer was asked to take photograph of all possible position of the students such that the two boys are separated.

One third of the photograph taken will be printed out. Find the total cost, in RM, if the costs of printing a picture is RM0.80.

[4 *marks*]

$^5P_5$ = All students arranged in a row

No of possible position

$= {}^5P_5 - ({}^4P_4 \times 2!)$

$= 72$

$^4P_4$ = assume the students are side by side and become 1 unit + with 3 girls

2! = The two boys can be arrange side by side

$$\text{Total cost} = \frac{1}{3} \times 72 \times 0.80$$

$$= 19.20$$

SPMRSM AND SPM QUESTIONS

SPM 2019 no. 22

QUESTION 32:

Diagram shows a four-digit passcode '0131' set by San on his smartphone.

He wants to reset the passcode such that the new passcode cannot consists of digit 1 followed by digit 3. How many different passcode can be formed?

[3 marks]

Number of passcode without any condition

$= 10 \times 10 \times 10 \times 10$

$= 10\,000$

Number of passcodes which cannot consist of digit 1 followed by digit 3

= Without condition – 1 followed by 3

$= 10\,000 - (100 + 100 + 100) + 1$

$= 9\,701$

Digit 1 followed by digit 3 :

i)

1	3		

$= 1 \times 1 \times 10 \times 10$

$= 100$

ii)

	1	3	

$= 10 \times 1 \times 1 \times 10$

$= 100$

iii)

		1	3

$= 10 \times 10 \times 1 \times 1$

$= 100$

iv)

1	3	1	3

(repetition)

$= 1$

What we need to know about COMBINATION?

1 Combinations of r objects chosen from n different objects at a time

ORDER OF ARRANGEMENT IS NOT IMPORTANT

2 Combinations with certain conditions

1. Combinations Of r Objects Chosen From n Different Objects At A Time

4.6 :Combinations Of r Objects Chosen From n Different Objects At A Time

$$^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

n =8

QUESTION 33:

Determine the number of ways to choose 3 flashcards from 8 different flashcards

r = 3

Number of ways

$$^{8}C_{3} = \frac{8!}{3!(8-3)!}$$

$$= 56$$

4.6 :Combinations Of r Objects Chosen From n Different Objects At A Time

r = 5

QUESTION 34:

A committee consists of 5 members will be chosen from 5 men and 4 women. Calculate the number of ways the committee can be formed

n = 9

The number of ways the committee can be formed

$= {}^{9}C_{5}$

$= \frac{9!}{5!(9-5)!}$

$= 126$

4.7 : Combinations With Certain Conditions

QUESTION 35:

r

n

Calculate the number of ways to choose a cleanliness committee that consists of 5 persons from 8 scouts and 7 cadets if the number of scouts must exceed the number of cadets.

Number of scouts = 8 **Number of cadets = 7**

Number of scouts needed	Number of cadets needed	Number of ways that a committee can be formed
5	0	${}^8C_5 \times {}^7C_0$
4	1	${}^8C_4 \times {}^7C_1$
3	2	${}^8C_3 \times {}^7C_2$

The number of committee that can be formed

$= {}^8C_5 \times {}^7C_0 + {}^8C_4 \times {}^7C_1 + {}^8C_3 \times {}^7C_2$

$= 1722$

4.7 : Combinations With Certain Conditions

QUESTION 36:

Packages of presents for a Children's Day celebration which consists of 6 items are to be chosen from 4 different boxes of geometrical sets and 5 different boxes of water colours. Calculate the number of different ways the presents can be packed if they cannot consist of more than 3 boxes of water colours.

Number of boxes of geometrical sets = 4

Number of boxes of water colours = 5

Number of boxes of geometrical sets needed	Number of boxes of water colours needed	Number of ways the presents can be packed
3	3	$^4C_3 \times {}^5C_3$
4	2	$^4C_4 \times {}^5C_2$
5	1	Impossible
6	0	Impossible

The number of different ways the presents can be packed

$= {}^4C_3 \times {}^5C_3 + {}^4C_4 \times {}^5C_2$

$= 50$

4.7 : Combinations With Certain Conditions

QUESTION 37:

There are 9 bangles, each with different colours. The bangles are divided equally among 3 girls. Calculate the number of different ways on how the division can be done.

Since 9 bangles are to be divided equally among 3 girls, each girls will get 3 bangles

The number of different ways on how the division can be done

$= {}^{9}C_{3} \times {}^{6}C_{3} \times {}^{3}C_{3}$

$= 1680$

From 9 bangles, 3 are chosen by the first girl

6 bangles left to be chosen by the second girl

The last 3 bangles wil be taken up by the third girl

SPMRSM, SPM AND FORECAST QUESTION (COMBINATION)

SPMRSM, SPM AND FORECAST QUESTIONS

QUESTION 38:

Solve ${}^{16}C_{r+3} = {}^{16}C_{7-r}$ (2 marks)

We compare r because n is same.

$$r + 3 = 7 - r \quad ✓$$
$$2r = 4$$
$$r = 2 \quad ✓$$

SPMRSM, SPM AND FORECAST QUESTIONS

QUESTION 39:

Given $3\left({}^{n}C_{4}\right) = 5\left({}^{n-1}C_{5}\right)$, find the value of ${}^{n}C_{9}$. (3 MARKS)

$$3\left[\frac{n!}{(n-4)!\,4!}\right] = 5\left[\frac{(n-1)!}{(n-1)-5)!\,5!}\right] \checkmark$$

$$\frac{3(n!)}{(n-1)!} = \frac{5\times(n-4)!\times 4!}{(n-6)!\times 5\times 4!}$$

$$\frac{3(n\times(n-1)!)}{(n-1)!} = \frac{(n-4)(n-5)(n-6)!}{(n-6)!}$$

$$3n = n^2 - 9n + 20$$
$$n^2 - 12n + 20 = 0$$
$$(n-10)(n-2) = 0 \checkmark$$
$$n = 10, n = 2$$

$${}^{n}C_{r} = \frac{n!}{(n-r)!r!}$$

Find ${}^{n}C_{9}$

So we choose n = 10

${}^{10}C_{9} = 10$ ✓

QUESTION 40:

Company XYZ is looking for 4 new employees among 7 candidates attending the interview. The company's policy does not allow husband and wife to work together. Given there is a couple of husband and wife amongst the candidates, find the number of different ways to select the new employees.

[3 *marks*]

$$\text{Number of different way} = {}^{7}C_{4} - ({}^{2}C_{2} \times {}^{5}C_{2})$$

To fulfil the requirement of 4 new employees

Choose 4 employee from 7 candidates

$= 25$

Husband and wife chosen

SPMRSM AND SPM QUESTIONS

SPM 2015 no. 21

QUESTION 41:

(a) State the value of ${}^{n}C_{0}$.

(b) Three students are to be selected from 5 boys and 4 girls to participate in a competition.
Find the number of different ways to choose the participants if
(i) all the three students are boys,
(ii) one boy and two girls are chosen.

[3 marks]

(a) ${}^{n}C_{0} = 1$

(b) i) Number of combinations

$= {}^{5}C_{3} \times {}^{4}C_{0}$

From 4 girls we choose 0 girls

$= 10$

From 5 boys we choose 3 boys

ii) Number of combination

$= {}^{5}C_{1} \times {}^{4}C_{2}$

From 4 girls we choose 2 girls

$= 30$

From 5 boys we choose 1 boys

SPMRSM, SPM AND FORECAST QUESTIONS

QUESTION 42:

A coach wants to choose 5 players consisting of 2 boys and 3 girls to form a badminton team. These 5 players are chosen from a group of 4 boys and 5 girls. Find

(a) the number of ways the team can be formed,
(b) the number of ways the team members can be arranged in a row for a group photograph, if the three girls sit next to each other.

[4 marks]

(a) Number of ways the team can be formed

$= {}^{4}C_{2} \times {}^{5}C_{3}$ ← From 5 girls we choose 3 girls

$= 60$

From 4 boys we choose 2 boys

(b) Number of ways the team members can be arranged in a row

$= {}^{3}P_{3} \times 3!$

$= 36$

${}^{3}P_{3}$: to arrange 2 Boys and girls(3 sit together)

3! = The girls interchange among themself

SPMRSM, SPM AND FORECAST QUESTIONS

QUESTION 43:

Dahlia has a home decoration shop. One day, Dahlia received 14 sets of cups from a supplier. Each set contained 6 pieces of cups of different colours.

(a) Dahlia choose 3 sets of cups at random to be checked. Find the number of different ways that Dahlia uses to choose those sets of cups.

(b) Dahlia takes a set of cups to display by arranging it in a row. Find the number of different ways the cups can be arranged such that the blue cup is not placed next to the red cup.

[4 marks]

(a) The number of different ways

$= {}^{14}C_3$

$= 364$

We choose 3 sets from 14 sets

(b) The number of different ways the cups can be arranged

$= {}^{6}P_6 - ({}^{5}P_5 \times 2!)$

$= 480$

${}^{6}P_6$: Arrangement of all 6 pieces of cups

${}^{5}P_5$: Arrangement of the other 4 cups and (blue n red cups)

2! : Blue and red cups interchange among them

SPMRSM, SPM AND FORECAST QUESTIONS

QUESTION 44:

A teacher wants to choose 7 students consisting of 3 boys and 4 girls to form a committee of science and Mathematics Club. These 7 students are chosen from 5 boys and 6 girls. Find the number of ways

(a) the committee can be formed,

(b) the committee can be arranged at a round table in the meeting if the four girls sit next to each other.

[4 marks]

(a)The committee can be formed

$= {}^{5}C_{3} \times {}^{6}C_{4}$ ← From 6 girls we choose 4 girls

$= 150$

From 5 boys we choose 3 boys

(b) No of ways

$= (4 - 1)! \times {}^{4}P_{4}$

$= 144$

(4-1)! : Arrangement of all 4 girls around a table

${}^{4}P_{4}$: interchange among the 4 girls themselves

SPMRSM, SPM AND FORECAST QUESTIONS

QUESTION 45:

13 stalks of flowers consisting of 3 red flower, 4 blue flowers and the remain are white flowers. Find the number of ways

(a) If a bouquet of flower is made from eight flowers only,

(b) The flowers will be arranged at a round table. Calculate the number of ways to arrange the flowers.

[4 marks]

(a)Number of ways

$= {}^{13}C_8$

$= 1287$

From 13 flower we choose 8 flowers only

(b) The number of ways to arrange the flowers

$= \frac{(13-1)!}{3!\times 4!\times 6!}$

$= 4620$

(13-1)! : from circular permutation formula (n-1)!

3! : identical red flower
4! : identical blue flower
6! : identical white flower


THANK YOU

Siri Jom Skor A+ Matematik Tambahan

SPM 2021

22 Aug — 8.00 pm – 10.00 pm
Sahlawati Zakaria | MRSM Kuala Krai
Functions

27 Aug — 8.00 pm – 10.00 pm
Norlela Sapari | MRSM Taiping
Quadratic Functions

31 Aug — 8.00 pm – 10.00 pm
Khairulbariah Khairuddin | MRSM Mersing
Systems of Equations

4 Sept — 8.00 pm – 10.00 pm
Hazlina Ahmad | MRSM Alor Gajah
Indices, Surds and Logarithms

10 Sept — 8.00 pm – 10.00 pm
Hasniza Ismail | MRSM Parit
Progressions

16 Sept — 3.00 pm – 5.00 pm
Rosdiana Sarju | MRSM Johor Bahru
Linear Law

24 Sept — 8.00 pm – 10.00 pm
Nur Suhaila Abu Bakar | MRSM Tumpat
Coordinate Geometry
New Speaker

26 Sept — 8.00 pm – 10.00 pm
Mohd Faizi Mamat | MRSM Gemencheh
Vectors

1 Oct — 8.00 pm – 10.00 pm
Abdul Hadi Azmi | MRSM Pengkalan Chepa
Solution of Triangles

8 Oct — 8.00 pm – 10.00 pm
Noraini Ismail | MRSM Transkrian
Index Numbers

10 Oct — 8.00 pm – 10.00 pm
Hariani Abidin | MRSM Kuching
Circular Measure

15 Oct — 8.00 pm – 10.00 pm
Erwan Hazreen Musa | MRSM Bentong
Differentiation

18 Oct — 8.00 pm – 10.00 pm
Mohamad Fauzi Razak | MRSM Kepala Batas
Integration
New Date

23 Oct — 8.00 pm – 10.00 pm
Muhamad Baginda Zainuddin | MRSM Batu Pahat
Kinematics of Linear Motion
New Date

30 Oct — 3.00 pm – 5.00 pm
Haziq Syazwan Sajali | MRSM Tun Mustapha
Trigonometric Function
New Date

5 Nov — 3.00 pm – 5.00 pm
Suhaila Sulong | MRSM Tun Dr. Ismail
Permutation and Combination
New Date

7 Nov — 8.00 pm – 10.00 pm
Norhafizah Mohamed Yusoff | MRSM K. Terengganu
Probability Distribution
New Date

17 Dis — 8.00 pm – 10.00 pm
Asniza Arshad | MRSM Tun Ghaffar Baba
Linear Programming

Anjuran Unit Matematik
Bahagian Pendidikan Menengah MARA

Sesi webinar *live* melalui Microsoft Teams